What is the radius of convergence for each power series?

1) $\sum \left(\frac{x}{2}\right)^n$ 2) $\sum \frac{x^n}{n!}$ 3) $\sum \frac{x^n}{n \cdot 2^n}$ 4) $\sum \frac{x^{2n}}{(2n)!}$

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1) $\lim_{n\to\infty} \left| \frac{\left(\frac{x}{2}\right)^{n+1}}{\left(\frac{x}{2}\right)^n} \right|$

$= \lim_{n\to\infty} \left| \frac{x^{n+1}}{2^{n+1}} \cdot \frac{2^n}{x^n} \right|$

$= \lim_{n\to\infty} \left| \frac{x}{2} \right|$

$= \left| \frac{x}{2} \right|$ $|x| < 2$

2) $\lim_{n\to\infty} \left| \frac{\frac{x^{n+1}}{(n+1)!}}{\frac{x^n}{n!}} \right|$

$= \lim_{n\to\infty} \left| \frac{x}{n+1} \right|$

$= 0 < 1$ $\therefore$ any fixed x, $|x| < \infty$

3) $\lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)\cdot 2^{n+1}} \cdot \frac{n \cdot 2^n}{x^n} \right|$

$= \lim_{n\to\infty} \left| \frac{nx}{2n+2} \right|$

$= \left| \frac{nx}{2n} \right|$

$= \left| \frac{x}{2} \right|$ $\therefore |x| < 2$

4) $\lim_{n\to\infty} \frac{x^{2(n+1)}}{(2(n+1))!} \cdot \frac{(2n)!}{x^{2n}}$

$= \lim_{n\to\infty} x^2 \cdot \frac{(2n)(2n-1)(2n-2)\cdots 1}{(2n+2)(2n+1)(2n)(2n-1)\cdots 1}$

$= \lim_{n\to\infty} \frac{x^2}{(2n+2)(2n+1)}$

$= \lim_{n\to\infty} \frac{x^2}{4n^2+6n+2}$

$= 0 < 1$ $\therefore$ any fixed x, $|x| < \infty$